Mastering the Dimensions of Correlations

Correlations are an essential component of a risk management and investment decision framework. They determine how risk aggregates across different asset classes and liabilities into portfolio or balance sheet risk. Unfortunately, correlations are very complex because they have many dimensions, and calibrating risk models to realistic correlation structures can be difficult and time-consuming. Practitioners often follow a "partial approach" in which they calibrate models on an application-specific subset of the correlation dimensions. However, such a partial approach is inefficient and inconsistent. In this paper, we discuss the various dimensions of correlations and illustrate how to master these with the help of well-designed and calibrated risk models.

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Correlations are an essential component of a risk management and investment decision framework. They determine how risk aggregates across different asset classes and liabilities into portfolio or balance sheet risk. For example, the amount of diversification in a portfolio is determined by the assumed asset correlation structure, and the Economic Capital of an insurance company is determined by the assumed correlations between assets and liabilities in worst-case scenarios.

Unfortunately, correlations are very complex because they have many dimensions. Calibrating risk models to realistic correlation structures can be difficult and time-consuming, especially as the dimensions of the models (*e.g.*, the number of economies and asset classes) increase. Practitioners are then often forced to follow a "partial approach" in which they calibrate models on an application-specific subset of the correlation dimensions (*e.g.,* for a particular investment horizon, for particular asset classes or for a particular economy).

Although understandable, such **a partial approach to correlation modeling is inefficient and inconsistent**. It is unavoidable that properly calibrating a risk model takes time, but having to calibrate a risk model multiple times to different correlation targets is inefficient. As pointed out by Varnell (2009), it also increases the risk of inconsistent decisions being taken throughout an organization. If multiple risk models are used, each calibrated to different correlation targets, there is no unifying underlying model that aggregates these correlations structures in a consistent way.

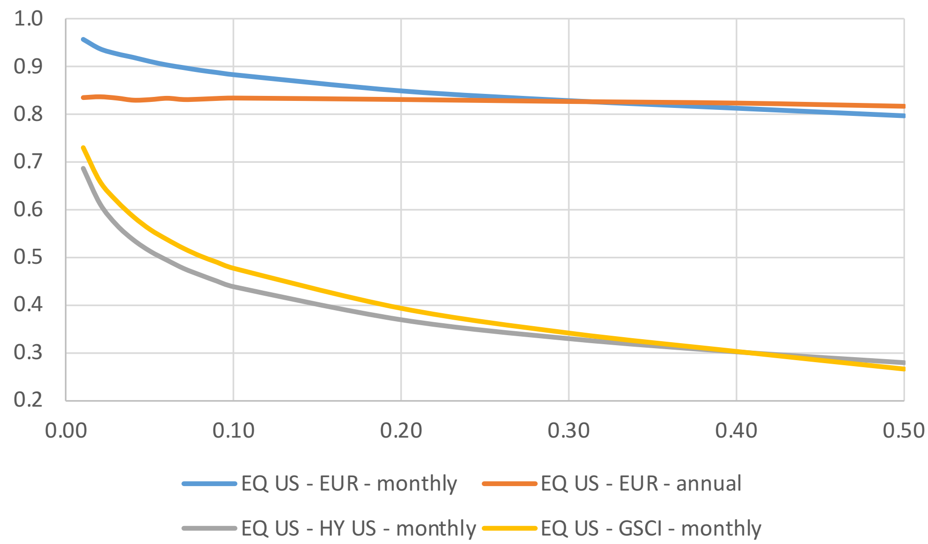
In this paper, we discuss the various dimensions of correlations and illustrate how these can be mastered with the help of well-designed and calibrated risk models.

THE POINT-IN-TIME DIMENSION: TAIL CORRELATIONS

Correlations are not defined by only the variable dimension (that is, the two economic or financial market variables for which their co-movement is of interest). A first additional dimension of correlations is time, in the sense of the economic and financial market conditions that hold at a particular point-in-time. Correlations between asset returns are typically higher in adverse economic and financial market conditions than in "normal" times. A prime and recent example is the pandemic outbreak of COVID-19 in the first quarter of 2020. As investors were trying to assess the impact of the outbreak on the global economy, correlations across asset classes and regions increased sharply and greatly reduced the much-sought diversification benefits.

This point-in-time dimension of correlations expresses itself in so-called tail correlation patterns. As illustrated in Figure 1, we typically see that **correlations increase**

Figure 1: Tail Correlations in the Left Part of Return Distributions[1]



as we move further into the negative parts of the return distributions. This holds for correlations between the returns on the same type of assets but in different regions (*e.g.*, between US and European equities), as well as between the returns on different types of assets in the same region (*e.g.*, between US equities and US High Yield corporate credits). Typically, such patterns of increasing tail correlations are stronger for higher frequency (*e.g.*, monthly or weekly) returns than for lower frequency (*e.g.*, quarterly or annual) returns.

To capture these important increasing tail correlation patterns, risk models are often based on Copulas. Unfortunately, **Copula-based approaches suffer from the "curse of dimensionality"**: the more variables need to be modelled, the harder it gets to calibrate these models properly. This can especially become problematic in a multi-asset class and multi-regional setting. For example, it may be feasible to properly calibrate the tail correlations between different regional equity benchmarks but challenging to simultaneously also properly calibrate the tail correlations between these equity benchmarks and High Yield corporate credits.

THE HORIZON DIMENSION

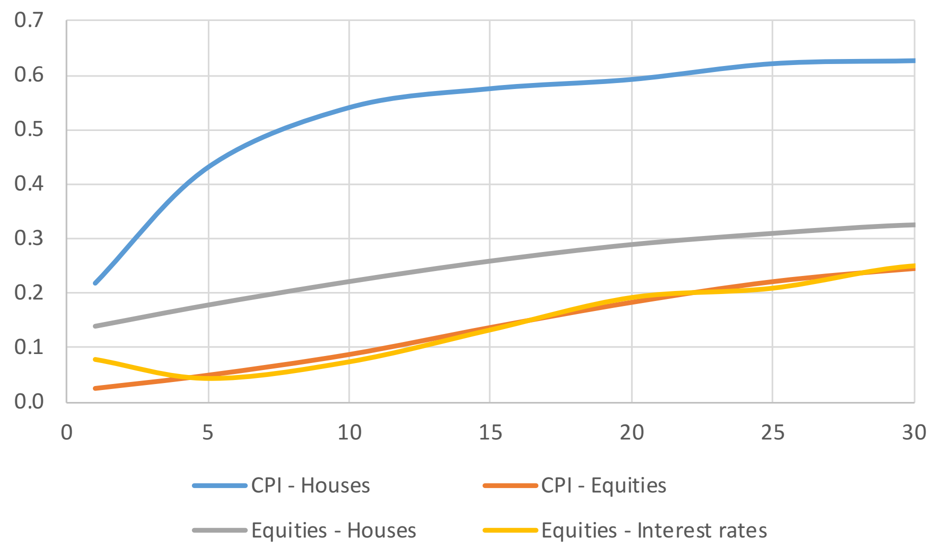
A second additional dimension of correlations is the horizon dimension: correlations can be different depending on the investment horizon. As illustrated in Figure 2, we typically see that **correlations tend to increase as the investment horizons extend**. More generally, not only correlations, but also expected returns, volatilities, and distributional shapes can vary with the investment horizon. General awareness of this so-called "term structure of risk and return", and its potential consequences for optimal asset allocations, were created by Campbell and Viceira (2005).

Short-term portfolio risk management models are often calibrated on relatively recent and relatively high-frequency data, focused on providing good quality assessments of short-term portfolio risk. However, such approaches are ill-suited for managing medium to long-term risk, for a horizon of one year up to several decades into the future. The reason is that **short-term models have a tendency to falsely "extrapolate" short-term correlations to long-term horizons**. If risk at multiple horizons matters, as it does in many organizations, practitioners often have to resort to multiple risk model calibrations, each one targeted at the correlations at a particular investment horizon, while accepting the inefficiencies and inconsistencies that result from such a partial approach.

CORRELATION UNCERTAINTY

To complicate matters even further, all the tail and horizon dependent correlation structures that we discussed

Figure 2: Correlations of US Cumulative Returns Across the Investment Horizon (in years)[2]



are also uncertain. They are typically estimated on empirical data in search for the "true" correlations. However, these true correlations of course do not exist, and even if they did, the amount and quality of the economic and financial data that we have at our disposal to base our estimates on, leaves a lot to be desired. As a result, **correlations are uncertain across all their dimensions**. As an example, Figure 3 illustrates the uncertainty about the important equity – interest correlation.

HOW TREND – CYCLE DECOMPOSITIONS CAN HELP

However, if correlations have many dimensions, and if it is so hard to capture all these dimensions in one single risk model, do organizations then just have to accept the inefficiencies and inconsistencies of working with multiple calibrated risk models? Well, the answer is actually no. With the help of well-designed

Figure 3: Uncertainty About 1-Year US Equities - Interest Rate Correlation[3]

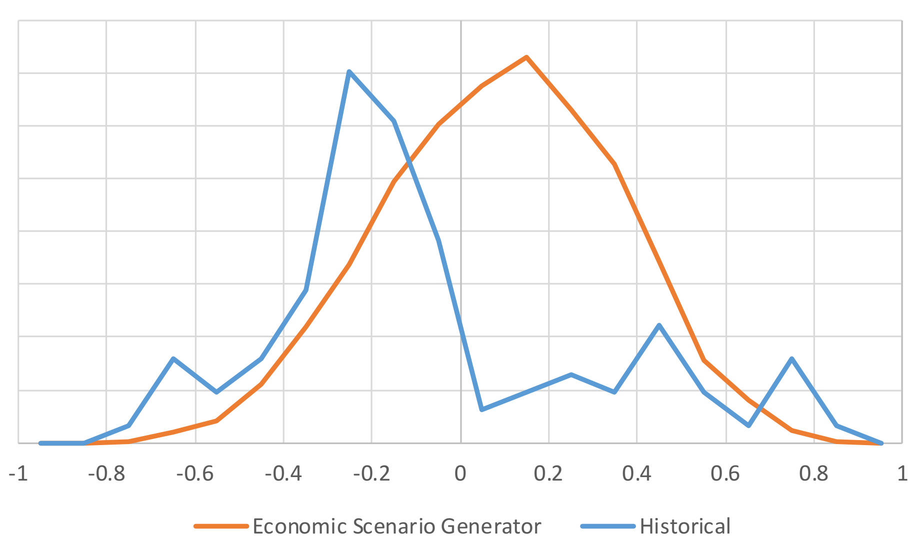
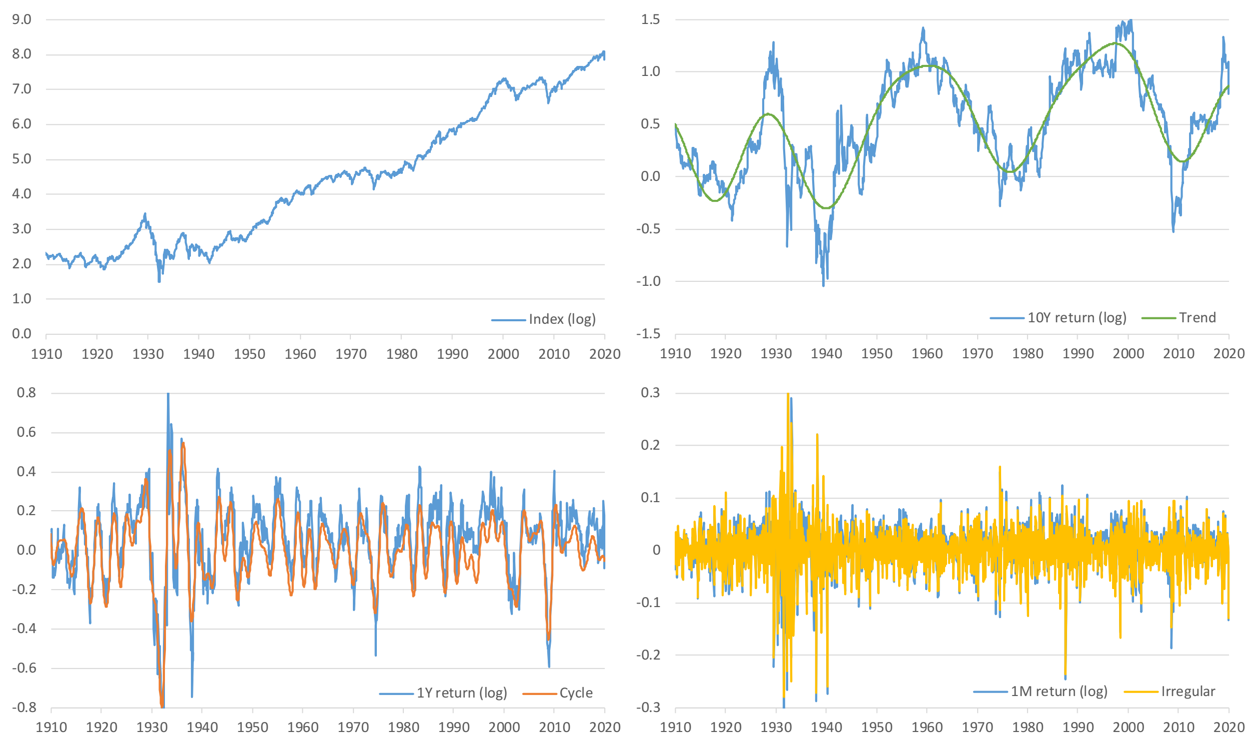


Figure 4: Trend - Cycle Decomposition of US Equities Total Return Index



and calibrated scenario-based risk models, it is possible to capture all the aforementioned dimensions of correlations efficiently and consistently in one single model calibration. Although a full exposition is beyond the scope of this paper, we do highlight some of the key components of what such a model can look like.

It is essential to introduce the concept of trend – cycle decompositions into a risk-modeling framework. Such decompositions can produce trend, cycle, and irregular components of benchmark indices that generate returns of different frequencies of this benchmark. Figure 4 illustrates how such components can approximately be interpreted as the decade, annual, and monthly returns of an index. By performing the same decomposition for all relevant economic and financial market variables, these components allow a model to "anchor" on the correlations for returns of different frequencies (and thereby also horizons).

This robust way of capturing the "term structure of risk and return" supports the use of a **single multi-horizon calibration** and thereby adds to the efficiency and consistency of enterprise-wide risk modeling. By complementing the use of trend – cycle decompositions with non-Normal dynamic factor modeling it also becomes possible to model **high-dimensional tail correlation structures** across economies and asset classes in an efficient and consistent way. For more details on how such a modeling approach works, see Steehouwer (2016).

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ENDNOTES

[1] Normal implied correlations based on Ortec Finance Economic Scenario Generator (ESG): correlation of a bivariate Normal distribution which corresponds to the measured Tail Dependence Coefficient (TDC) per quantile (threshold) in the left part of the distributions, where the TDC corresponds to the probability that one margin exceeds a threshold under the condition that the other margin exceeds a threshold. See e.g. Frahm et al. (2005).

[2] Based on Ortec Finance Economic Scenario Generator (ESG): correlations between cumulative (annualized) US CPI, geometric US equity returns, geometric US house price returns and the 10-year US Government yield, calculated on investment horizons of 1 to 30 years.

[3] *Orange*: histogram of correlation between annual returns on US equities and annual changes in 10-year US Government yield, calculated on 10,000 scenarios of 15 years from Ortec Finance Economic Scenario Generator (ESG). *Blue*: comparative histogram of historical correlations based on 105 rolling 15 year samples between 1900 and 2019. Note that the positive correlations of the last 20 years are weighted more heavily in the ESG model results.