Practical Guideline for Funding/Solvency Ratio Attribution

The funding ratio is a key measure used by Pension Funds the world over, and has a counterpart in the insurance world as the solvency ratio. This article gives a practical overview of how to explain the changes in this ratio over time. Within the article, we will use the terminology "funding ratio," but the methodological framework outlined can also be applied to the solvency ratio.

Both ratios will indicate the value of assets currently owned relative to the future liabilities that need to be funded by those assets. For the purposes of the ratio, future liabilities are discounted to their present value. However, it is uncertain whether the assets will meet the expected return implied by the discount rate. As these ratios indicate the likelihood that future liabilities can indeed be met, they are key health indicators for life insurers (solvency ratio) and defined benefit pension schemes (funding ratio).

In the article, we will first give a high-level overview of the suggested methodology, followed by its application to a specific illustrative case. We will explore how the methodology may be applied under different conditions, and conclude with a discussion on applicability. In the appendix, a more detailed description of the methodology is given.

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OUTLINE

The change in funding ratio is derived from the growth rate in both assets and liabilities. The geometric change in funding ratio is equal to the geometric excess growth of assets over liabilities.

$$\frac{FR_{end}}{FR_{start}} = \frac{(A_{end}/L_{end})}{(A_{start}/L_{start})} = \frac{A_{end}/A_{start}}{L_{end}/L_{start}} =$$

$$\frac{1 + A_{growth}}{1 + L_{growth}} = 1 + XS_{growth(GEO)}$$

Therefore, funding ratio attribution is more naturally solved using a geometric attribution framework (Geenen and Vermeulen, 2011). However, arithmetic frameworks are more widespread and will also provide useful insights. To explain the arithmetic funding ratio change, the arithmetic excess in growth rate needs to be multiplied[1] by a scaling factor $\frac{A_{start}}{L_{end}}$:

$$XS_{growth(ARITH)} \cdot \frac{A_{start}}{L_{end}} = \left(\frac{A_{end}}{A_{start}} - \frac{L_{end}}{L_{start}}\right) \cdot$$

$$\frac{A_{start}}{L_{end}} = \frac{A_{end}}{L_{end}} - \frac{A_{start}}{L_{start}} = FR_{end} - FR_{start}$$

Within this article we'll show both the geometric and arithmetic methodologies to decompose funding ratio changes.

We would like to decompose the change in funding ratio over a period into three main components.

1. Risk factors that are active decisions (*e.g.*, the tactical exposure to certain asset classes).

2. Risk factors that we expect to hedge (*e.g.*, currency exposure mismatch between assets and liabilities).

3. Risk factors that can't be hedged (at reasonable

costs) (*e.g.*, an update in life expectancy tables).

Our natural framework of choice would be a macro attribution (Geenen, Heemskerk, and Heerema, 2001) which decomposes the excess into its main decisions. However, the funding ratio change will not be decomposed into decisions alone, as external risk factors also contribute to the funding ratio change.

Depending on the regulatory regime and investment beliefs, a particular risk contributor may be assigned differently across the above categories. The most important driver is the regulatory requirement concerning the discount rate used for future liabilities. Another important step is transforming the discount rate into an investible benchmark.

Under certain regimes, this transformation can be a daunting task. In this article, we will assume that liabilities are discounted by the ex-ante expected return of the strategic asset allocation (ex-ante SAA, which is often determined within an Asset Liability Management study). With this assumption, the actual investible benchmark will then be the ex-post realized return of the same SAA.

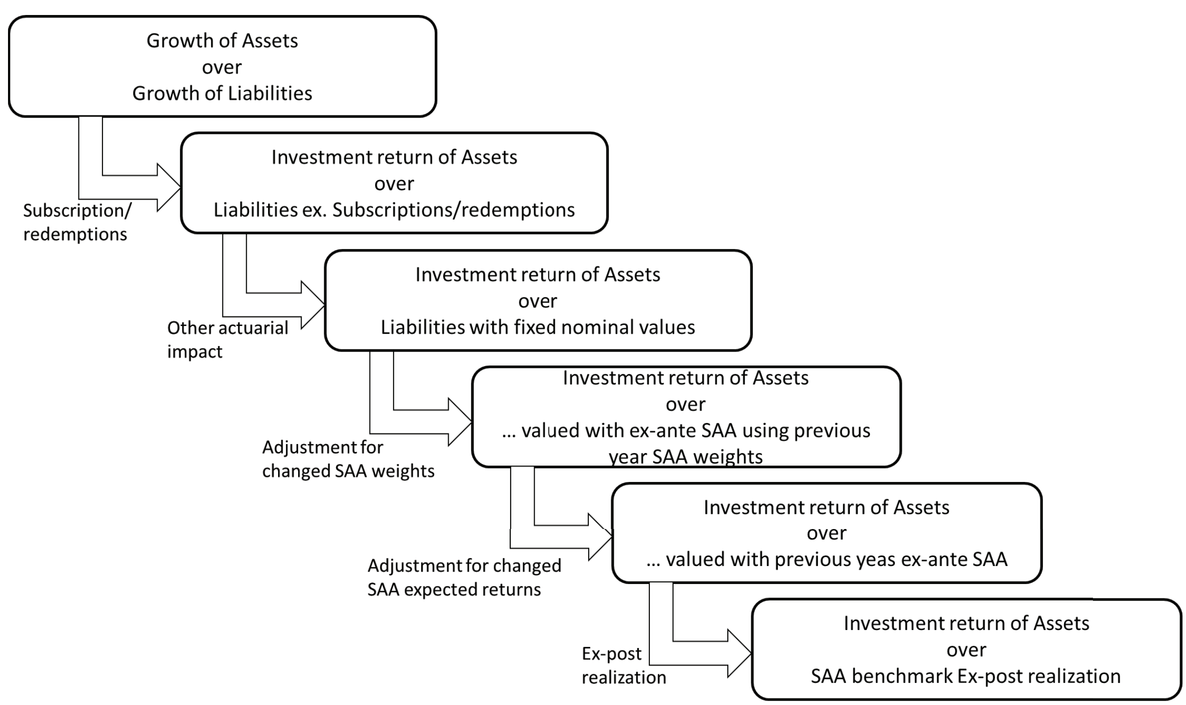
Depending on the chosen method for discounting the liabilities and the investment beliefs on how to transform this into an investible benchmark, the funding ratio change will be decomposed into different reporting components.[2]

We will model the growth rates of assets and liabilities as "investment returns" for the purposes of our formulas, but for the avoidance of doubt, they are not. Most importantly, external cash flows (subscriptions/redemptions) will impact their growth, but they are not part of the investment return. Nevertheless, external cashflows have an impact on the overall funding ratio, so they must be accounted for as a source of change. Finally, there are other actuarial risk factors like the updating of the life expectancy table, which affects the liability valuations and therefore also the ratio.

We have now introduced the basic concepts for a model of funding attribution. To separate the impact of several risk factors, one by one, we will have to value the liabilities under different assumptions of immunized[3] risk factors. Please note that the order in which we do this may be subject to debate though has arguably only limited impact.

For our case of ex-ante SAA discounting, we may construct the following layers of analysis (see Figure 1).

Figure 1: Layers of Analysis



At the bottom, we come to the traditional starting point for the analysis of the portfolio against an investible benchmark, the SAA. Further down, our actual investment decisions can be measured. These are obviously important, however, since this aspect is already substantially covered in existing literature, we will not consider these further here. In the appendix, we'll explain how the impact of all separate investment decisions may be tied to a macro attribution framework.

The layer that may need more consideration is the correction for the SAA expected returns. For most asset classes, a revision of the expected return will be modeled as a step shock at a specific date, for example when the model generating the expectations is recalibrated. However, if yields decline, this would be bad for the future SAA expected returns, but good for current (ex-post) realization on fixed income. If we didn't adjust the SAA expected return for fixed income on the same days as we observe the price changes, this would be a spurious source of negative correlation. Especially, when one monitors a duration overlay mandate, it will be key to re-evaluate liabilities to these adjusted expected rates of return on a frequent (daily) basis.

CASE

In our case, the funding ratio changes from 137.5 to 130, with the following detail data:

Table 1: Period Overview

	Start	End
Assets	110	130
Liabilities	80	100
Funding ratio	1.375	1.30

The changes in the value of the assets were due to the following components:

Table 2: Period Results on Assets

	delta
External cashflows	10
Investment return	10

By valuing the liabilities before and after several shocks, we can split up the growth in liabilities due to the following factors (see Table 3).

Table 3: Period Results on Liabilities

	Valuation	delta
At start of period	80	
With external cashflows	89	9
After actuarial impact	94	5
After actuarial & SAA mix changes impact	92	-2
After actuarial & SAA expected return impact	95	3
At end of period	100	-5

Based on this data, we can calculate returns for both the assets and liabilities on several layers (see Table 4).

These returns may be used directly in a macro decomposition framework (see Table 5).

As the arithmetic decomposition explains the arithmetic excess growth (-6.8%), its values need to be multiplied by the factor $\frac{A_{start}}{L_{end}} = 1.1$ to explain the funding ratio change (-7.5%). The geometric effects may be taken without further modification. This will result in the following attribution effects (see Table 6).

In our example, the funding ratio dropped from 137.5 to 130 points due to the following causes:

1. External cashflows (net inflows) reduced our funding ratio by 2.4 points as they were paid in just over par value while our funding ratio was well above 100.

2. Actuarial updates led to a decrease in the funding ratio of 6.9 points.

3. Our future expected returns were reduced because

Table 4: Calculated Returns

	Assets				Liabilities			
	Start Value	End Value	CF	Return	Start Value	End Value	CF	Return
Growth in assets vs growth in liabilities	110	130	0	18.2%	80	100	0	25.0%
External cash flows assets			-10					
External cash flows liabilities							-9	
Cleaned for external cashflows	110	130	10	9.1%	80	100	9	13.8%
Actuarial impact			0				-5	
Cleaned for actuarial impact	110	130	10	9.1%	80	100	14	7.5%
Impact of SAA asset mix changes			0				2	
Cleaned of SAA asset mix changes	110	130	10	9.1%	80	100	12	10.0%
Impact of SAA expected returns			0				-3	
Cleaned all SAA changes	110	130	10	9.1%	80	100	15	6.3%
Impact of ex-post realization			0				5	
Investment return vs SAA	110	130	10	9.1%	80	100	10	12.5%

Table 5: Calculated Effects

Input		Arithmetic		Geometric	
Asset	Liabilities	XS	Delta	XS	Delta
18.2%	25.0%	-6.8%		-5.5%	
9.1%	13.8%	-4.7%	**-2.2%**	-4.1%	**-1.4%**
9.1%	7.5%	1.6%	**-6.3%**	1.5%	**-5.5%**
9.1%	10.0%	-0.9%	**2.5%**	-0.8%	**2.3%**
9.1%	6.3%	2.8%	**-3.8%**	2.7%	**-3.4%**
9.1%	12.5%	**-3.4%**	**6.3%**	**-3.0%**	**5.9%**

Table 6: Funding Ratio Decomposition

	Arithmetic	Geometric
Funding ratio change	-7.5%	-5.5%
External cashflows	-2.4%	-1.4%
Actuarial impact	-6.9%	-5.5%
Ex-ante SAA discounting impact	-1.4%	-1.2%
SAA Asset mix changes	2.8%	2.3%
SAA expected return changes	-4.1%	-3.4%
Ex post realization	6.9%	5.9%
All Investment decisions	-3.8%	-3.0%

we became more pessimistic on future asset class returns. This was partly offset by a more aggressive SAA asset-mix. Altogether, the changes in future anticipated returns reduced the funding ratio by 1.4 points.

4. It turned out to be an above average investment year (ex post realization was well above ex ante expectation for the SAA), which increased the funding ratio by 6.9 points. However, the investment decisions that were taken beyond the SAA reduced the funding ratio by 3.8 points.

DISCUSSION ON APPLICABILITY

The methodology is described in terms of a defined benefit (DB) pension scheme funding ratio. The methodology is also applicable to life insurance solvency ratio changes. That being said, for solvency ratio attribution, the following points should be considered:

- The coverage ratio typically deviates further from 100%, which amplifies the differences between arithmetic and geometric methods. The geometric method may become more preferable as a result.

- By actively selling more/fewer policies, there is some discretion in the subscription/redemption cash flows. One could consider splitting out cash flows from new business (risk factor with active decision) vs. cash flows from continued contracts (risk factor that is a given).

- Regulatory discounting frameworks may be different (typically stricter for Insurers).

Another extension we would like to consider is applying the framework to defined contribution (DC) pension schemes. In DC schemes, the funding ratio itself is not applicable. However, changes in the expected future benefits for the capital invested at the start of a period are sensitive to similar risk factors. The provided framework is most applicable for DC schemes that are expected to provide a steady base income stream upon retirement, rather than DC schemes that manage surplus capital. Under those circumstances, from the perspective of beneficiaries, investments in DC schemes serve broadly the same purpose as they would in DB schemes and should thus be managed in a similar manner. That said, we can see some core differences in the attribution framework:

- The arithmetic analysis is more likely to give accurate results because of two characteristics:

    - The "coverage ratio" is "pegged" at the start of the analysis to 100 percent. This reduces one major source that needs additional scaling in the arithmetic analysis.

    - If one assumes that the "price" at which participants may buy future benefits or receive payouts is fair, the subscription and redemptions are expected to make only marginal changes. This reduces the impact of large monthly/yearly shocks on arithmetic smoothing methods.

- The regulatory discounting framework is typically more relaxed. This means that we may be able to better define our discounting policy such that it translates into an investible benchmark with less uncontrollable risk.

CONCLUSIONS

We have given a practical approach in attributing the change of the funding ratio for a defined benefit pension fund to the different sources impacting the funding ratio. We have introduced an arithmetic and geometric framework for funding ratio attribution. Finally, we discussed how the framework can be applied to life insurers, as well as defined contribution pension schemes.

APPENDIX - METHODOLOGY

<u>Beyond the Marco Attribution Method</u>

To come to the details of the macro attribution model output, we will use a multi allocation layer (top-down) Brinson model. The idea is that every "risk factor" is modeled as a separate allocation layer. For each layer we will calculate its own "allocation" effect. The first allocation effect would be the contribution of the subscription/redemptions, the second level the actuarial impact and so on. The general form for the top-down formulas are as follows:

$$Allocation := (w_{pf} - w'_{bm}) \,.\, (r_{bm} - R_{bm})$$
$$Selection := w'_{bm}.\left(r_{pf} - r_{bm}\right)$$
$$Interaction := (w_{pf} - w'_{bm}) \,.\, \left(r_{pf} - r_{bm}\right)$$

with:

$$w'_{bm} := \frac{w_{pf\ to\ top}}{w_{pf\ to\ parent}} . w_{bm\ to\ parent}$$

However, the multi-level Brinson model assumes a consistent benchmark. That is, the benchmark on a higher level is completely constructed from the segment benchmarks from lower levels. Despite ex-ante and ex-post SAA benchmarks being very similar in their description, neither will be a construction of the other. Using them directly would violate the assumptions of the model.

Instead, we will model the higher-level benchmark (ex-ante SAA) as being made up of two components. The first component is a 100% weight in the lower-level benchmark (ex-post SAA). The second component will be a virtual asset. The virtual asset should be weightless; however, it contributes the difference between ex-ante and ex-post, such that on total level the ex-ante SAA matches up.

Unfortunately, we are still not there yet. Weightless contributors are not covered in typical Brinson models as they assume that the return can be decomposed into the sum over all segments of the product of weight and return within the segment, that is $R = \sum_s w_s \cdot r_s$. We will therefore need to expand the typical formula set. Fortunately, we can assign the contribution of weightless contribution segments to either the allocation or selection effect. In our example case, the allocation effect is the most obvious choice.[4]

Another place where weightless contributors may have an impact is the Brinson-Fachler opportunity cost term R_{bm}. In general, we may assume that weightless benchmark segments are not part of the investible benchmark, so we should clean the total benchmark return of these weightless contributing segments before using them in the opportunity cost term. The formulas in our adjusted Multi level Brinson-Fachler model will then become:

$$Allocation := (w_{pf} - w'_{bm}) \,.\, (r_{bm} - R'_{bm}) + wlcb_{pf} - wlcb_{bm}$$
$$Selection := w'_{bm}.\left(r_{pf} - r_{bm}\right)$$
$$Interaction := (w_{pf} - w'_{bm}) \,.\, \left(r_{pf} - r_{bm}\right)$$

with:

$$w'_{bm} := \frac{w_{pf\ to\ top}}{w_{pf\ to\ parent}} . w_{bm\ to\ parent}$$
$$R'_{bm} := \sum_{all\ allocation\ segments} w_{bm\ to\ parent} \cdot r_{bm}$$
$$wlcb_x := \begin{cases} 0, & w_x \neq 0 \\ contribution_{x\ to\ top}, & w_x = 0 \end{cases}$$

Table A1: Calculated Returns (=Table 4)

Given the set of returns as calculated in the main article:

	Assets				Liabilities			
	Start Value	End Value	CF	Return	Start Value	End Value	CF	Return
Growth in assets vs growth in liabilities	110	130	0	18.2%	80	100	0	25.0%
External cash flows assets			-10					
External cash flows liabilities							-9	
Cleaned for external cashflows	110	130	10	9.1%	80	100	9	13.8%
Actuarial impact			0				-5	
Cleaned for actuarial impact	110	130	10	9.1%	80	100	14	7.5%
Impact of SAA asset mix changes			0				2	
Cleaned of SAA asset mix changes	110	130	10	9.1%	80	100	12	10.0%
Impact of SAA expected returns			0				-3	
Cleaned all SAA changes	110	130	10	9.1%	80	100	15	6.3%
Impact of ex-post realization			0				5	
Investment return vs SAA	110	130	10	9.1%	80	100	10	12.5%

Table A2: Calculated Effects (Arithmetic)

We may calculate the following effects:

	Effects						
	Allocation effects					Selection	Total
	L1	L2	L3	L4	L5		
Growth in assets vs growth in liabilities	-2.2%	-6.3%	2.5%	-3.8%	6.3%	-3.4%	-6.8%
External cash flows assets	9.1%						9.1%
External cash flows liabilities	-11.3%						-11.3%
Cleaned for external cashflows	0.0%	-6.3%	2.5%	-3.8%	6.3%	-3.4%	-4.7%
Actuarial impact		-6.3%					-6.3%
Cleaned for actuarial impact		0.0%	2.5%	-3.8%	6.3%	-3.4%	1.6%
Impact of SAA asset mix changes			2.5%				2.5%
Cleaned of SAA asset mix changes			0.0%	-3.8%	6.3%	-3.4%	-0.9%
Impact of SAA expected returns				-3.8%			-3.8%
Cleaned all SAA changes				0.0%	6.3%	-3.4%	2.8%
Impact of ex-post realization					6.3%	0.0%	6.3%
Investment return vs SAA					0.0%	-3.4%	-3.4%

Which explains the arithmetic excess in growth. We still have to multiply these numbers with the factor

$$\frac{A_{start}}{L_{end}} = 1.1$$

to explain the arithmetic funding ratio change of -7.5 points. With this factor and a pivot of the top row, we obtain the decision attribution style output in the main article (see Table A3).

GEOMETRIC ATTRIBUTION

Personally, I am not a strong proponent of either arithmetic or geometric methods or specific smoothing operations in general. I have never seen a practical case in which such choices influence the qualitative conclusions that could be drawn from the resulting report. However, as discussed previously, the funding ratio attribution is by its nature a geometric problem, so we will extend the method to a geometric scheme.

We will provide a framework that can be applied using any method that can transform (smooth) any set of arithmetic effects (not summing to zero) into geometric effects that accumulate to a given geometric total effect. In our case we'll illustrate this using the arithmetic

Table A3 Funding Ratio Decomposition: Arithmetic

Total funding ratio change	-7.5%
External cashflows	-2.4%
Actuarial impact	-6.9%
Ex-ante SAA discounting impact	-1.4%
SAA Asset mix changes	2.8%
SAA expected return changes	-4.1%
Ex post realization	6.9%
All Investment decisions	-3.8%

smoothing method by (Cariño, 1999) to resolve to geometric effects. This is mainly for practical reasons, as it is a methodology that can be expressed in relatively simple formulae. To be sure, the Cariño sums mentioned in the article can be transformed into geometric effects by applying $e^{CARINO\_SUM} - 1$ as outlined in the picture below (Figure A1).

Please note that the two pivotal returns that are chosen as inputs for the K factor play the crucial role as these will set the frame of reference to which geometric excess is being smoothed.

When one would use a single K-factor for the entire analysis, the allocation will not aggregate to the geometric allocation effect. To overcome this, we'll have to use multiple K-factors, one to smooth allocation effects and one to smooth the remainder. Using A as the total arithmetic allocation effect

$$A := \sum_{S} Alloc_{S(ARI)} ,$$

the K-factor for allocation smoothing is defined by

$$K(R_{BM} + A, R_{BM}) := \frac{LN(1 + R_{BM} + A) - LN(1 + R_{BM})}{A}$$

Using the following definition of geometric allocation effects:

$$Alloc_{S(GEO)} := EXP\big(Alloc_{S(ARI)}.K(R_{BM} + A, R_{BM})\big) - 1$$

We can show that this aggregates up to the geometric excess of the allocation decisions alone.

$$\prod_{S} 1 + Alloc_{S(GEO)} =$$

$$\prod_{S} EXP(\,Alloc_{S(ARI)}.K(R_{BM} + A, R_{BM})) =$$

$$EXP(\sum_{S} Alloc_{S(ARI)}.K(R_{BM} + A, R_{BM})) =$$

$$EXP(K(R_{BM} + A, R_{BM}).A) =$$

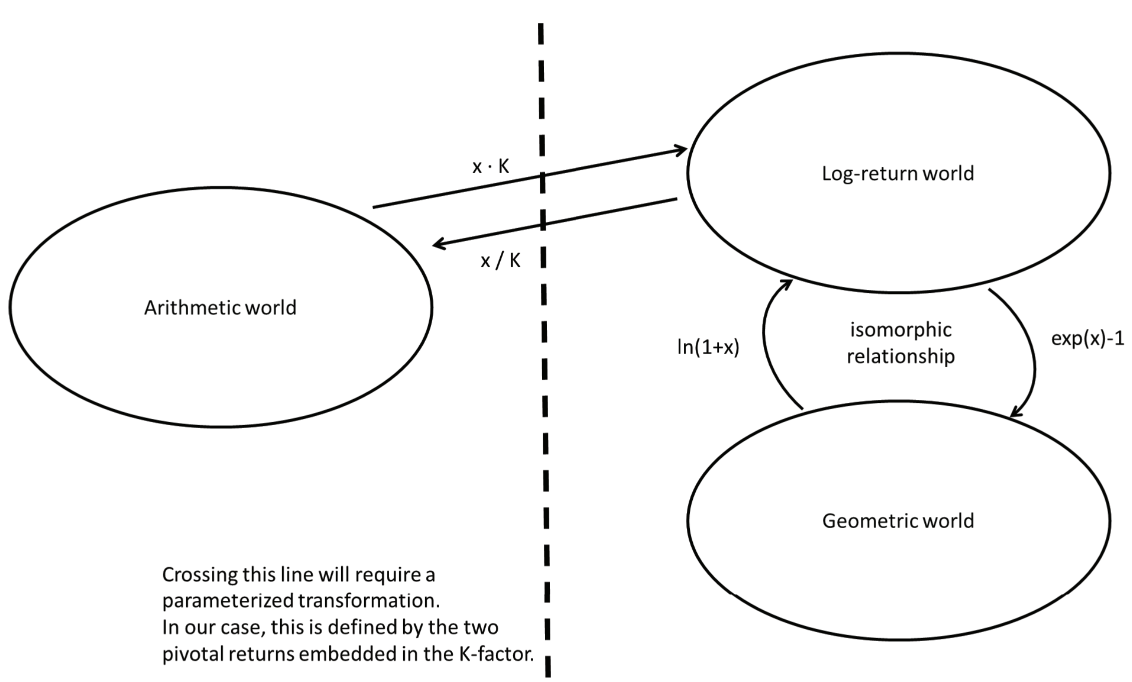
$$EXP\left(\frac{LN(1 + R_{BM} + A) - LN(1 + R_{BM})}{A}.A\right) =$$

$$EXP\big(LN(1 + R_{BM} + A) - LN(1 + R_{BM})\big) = \frac{1 + R_{BM} + A}{1 + R_{BM}}$$

For the remainder of the effects (selection + interaction) we'll use the K-factor defined by

$$K(R_{PF}, R_{BM} + A) := \frac{LN(1 + R_{PF}) - LN(1 + R_{BM} + A)}{R_{PF} - R_{BM} - A}$$

Figure A1: Relation Arithmetic and Geometric



In a similar way as with the allocation effects, it can be shown that using this K-factor, the arithmetic effects that explain $R_{PF} - R_{BM} - A$ can be transformed into explaining the geometric excess

$$\frac{1+R_{PF}}{1+R_{BM}+A} - 1.$$

We can assign K-factors for deeper layer allocation levels as long as we have an opinion on the order in which allocation decisions are taken. This may become problematic however when having to compare regional allocation within equity segments to investment grade allocation within the fixed income asset class. Thus, at some point in the decision hierarchy, all decisions have to be treated with the same K-factor. Within our simplified example, we do not have this issue, so we can treat each allocation level in order, leading to the following set of K-factors:

$$K_{ALLOC}(level) \coloneqq K\left(R_{BM} + \sum_{l=1}^{level} A_l\,, R_{BM} + \sum_{l=1}^{level-1} A_l\right)$$

$$K_{SELEC} \coloneqq K\left(R_{PF},\ R_{BM} + \sum_{l=1}^{MAX\_LEVEL} A_l\right)$$

GEOMETRIC ATTRIBUTION AND WEIGHTLESS CONTRIBUTIONS

Portfolio weightless contributions may pose an issue to several[5] geometric smoothing schemes. Suppose we have an equal arithmetic contribution of external cash flows to assets and liabilities, such that in the arithmetic analysis, the impact is zero. However, unlike the arithmetic excess, the geometric excess changes when both sides of the analysis receive the same contribution if

Table A4: Example Zero Arithmetic Effect

	Assets	Liabilities	Excess Arithmetic	Excess Geometric
Growth assets vs growth liabilities	18.2%	25.0%	-6.8%	-5.5%
Cleaned for external cashflows	9.1%	15.9%	-6.8%	-5.9%

Table A5: K-Factor Calculation

	Returns Assets	Returns Liabilities	Returns Excess	K-factor calculation pivotal returns 'PF'	'BM'	K-factor
Growth in assets vs growth in liabilities	18.2%	25.0%	-5.5%			
External cash flows assets				18.2%	9.1%	0.88
External cash flows liabilities				13.8%	25.0%	0.84
Cleaned for external cash flows	9.1%	13.8%	-4.1%			
Actuarial impact				7.5%	13.8%	0.90
Cleaned for actuarial impact	9.1%	7.5%	1.5%			
Impact of SAA asset mix changes				10.0%	7.5%	0.92
Cleaned of SAA asset mix changes	9.1%	10.0%	-0.8%			
Impact of SAA expected returns				6.3%	10.0%	0.92
Cleaned all SAA changes	9.1%	6.3%	2.7%			
Impact of ex-post realization				12.5%	6.3%	0.91
Investment return vs SAA	9.1%	12.5%	-3.0%	9.1%	12.5%	0.90

there are some other contributors as well. Please find an example below (Table A4).

We can solve for this by applying a different K-factor to the allocation effects that originate from portfolio weightless positions. Let A_PFWL denote the portion of the allocation effect due to portfolio weightless positions.

$$K_{A\_PFWL}(level) \coloneqq$$
$$K\left(R_{PF} - \sum_{l=1}^{level-1} \text{A\_PFWL}_l\, , R_{PF} - \sum_{l=1}^{level} \text{A\_PFWL}_l\right)$$

Since we have already taken out all portfolio weightless allocation contributions of the portfolio return to smooth to, the deepest level selection K factor also has to be modified to account for this:

$$K_{SELEC} \coloneqq K\left(R_{PF} - \sum_{l=1}^{MAX\_LEVEL} \text{A\_PFWL}_l\, ,\ R_{BM} + \sum_{l=1}^{MAX\_LEVEL} A_l - \text{A\_PFWL}_l\right)$$

Given our example before, we'll end up with the following K-factor calculations (see Table A5).

Applying $e^{K*Arithmetic\_effect} - 1$, we'll find the following geometric effects (see Table A6).

Which can be pivoted into a macro attribution style output as shown in the main article (see Table A7).

Table A6: Calculated Effects (Geometric)

	Effects						
	Allocation/Decision effects					Selection	Total
	L1	L2	L3	L4	L5		
Growth in assets vs growth in liabilities	-1.4%	-5.5%	2.3%	-3.4%	5.9%	-3.0%	-5.5%
External cash flows assets	8.3%						8.3%
External cash flows liabilities	-9.0%						-9.0%
Cleaned for external cash flows		-5.5%	2.3%	-3.4%	5.9%	-3.0%	-4.1%
Actuarial impact		-5.5%					-5.5%
Cleaned for actuarial impact			2.3%	-3.4%	5.9%	-3.0%	1.5%
Impact of SAA asset mix changes			2.3%				2.3%
Cleaned of SAA asset mix changes				-3.4%	5.9%	-3.0%	-0.8%
Impact of SAA expected returns				-3.4%			-3.4%
Cleaned all SAA changes					5.9%	-3.0%	2.7%
Impact of ex-post realization					5.9%	0.0%	5.9%
Investment return vs SAA						-3.0%	-3.0%

Table A7: Funding Ratio Decomposition (Geometric)

Total funding ratio change (geometric)	-5.5%
External cashflows	-1.4%
Actuarial impact	-5.5%
Ex-ante SAA discounting impact	-1.2%
SAA Asset mix changes	2.3%
SAA expected return changes	-3.4%
Ex post realization	5.9%
All Investment decisions	-3.0%

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ENDNOTES

[1] In the perspective of multi-period attribution, we have two options. We can apply the factor every calculation interval and sum the results together, this will result in a path dependent smoothing in which periods with larger liabilities growth and/or ratios receive more weight than others do. The other option is to apply the factor only at reporting time after an arithmetic smoothing method of choice already has been applied.

[2] When the discount rate(s) do not naturally translate into an investible benchmark (like in our example), the behavior of the discount rates needs to be approximated by a proxy benchmark that is investible instead. It will take economical modelling decisions(which one may separately report) to set up the proxy. Finally there will also be a residual between the behavior and the proxy and the actual discounting rates which also needs to be labeled separately.

[3] Immunized: as if the particular risk factor had no impact on the valuation.

[4] In general, however, there may be cases where selection is more appropriate, for example when one observes a late tax rebate for a position that was previously closed. At the same time, it may be impractical to label all occurrences whenever they occur.

[5] Please note that there are some geometric smoothing methods that allow to smooth an arithmetic total effect of 0 to a non-zero geometric total effect. These methods will not need this additional trick, however such methods come with the disadvantage that any arithmetic zero effect may be transformed into a non-zero geometric effect.